

A SYSTEM OF NOISE REDUCTION EMPLOYING
TWO AMPLITUDE MODULATED WAVES

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A SYSTEM OF NOISE REDUCTION EMPLOYING
TWO AMPLITUDE MODULATED WAVES

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TABLE OF CONTENTS

	PAGE
Acknowledgment.....	iii
Summary.....	1
Frequency Modulation.....	4
Pulse-Time Modulation System.....	6
Pulse-Width Modulation System.....	6
Amplitude Modulation System.....	7
Theory of the Two-Carrier Amplitude Modulation System.....	8
Two-Carrier System of Amplitude Modulation.....	8
Frequency and Power Relations.....	10
Noise Reducing Properties.....	15
Random Noise in Case 1.....	16
Random Noise in Case 2.....	17
The Effect of Impulse Noise on Cases 1 and 2.....	23
Apparatus and Experimental Data.....	32
General Description of the System.....	32
Description of the Transmitters.....	32
Description of the Receivers.....	36
Experimental Techniques and Data.....	39
Objectives of the Experiment.....	39
Observed Noise Reduction.....	39
Observed Increase in Received Signal.....	41
BIBLIOGRAPHY.....	44
APPENDIX.....	45

LIST OF FIGURES

FIGURE		PAGE
1.	The Amplitude Modulation Process.....	12
2.	Modulated Waves Emanating from Each Transmitter.....	13
3.	Modulating Wave with Axes for Resolution and Lines Indicating Crest, Root-Mean-Square, and Average Values..	13
4.	Carrier and Side-Band Currents and Powers Represented as the Lines of a Frequency Spectrum.....	14
5.	Waveforms of the Two-Carrier System.....	20
6.	Form of the Rectangular Wave, Axes for Resolution of the Wave into a Fourier Series, and Dimensions.....	26
7.	Fourier Series Coefficients.....	27
8.	Summation of Components from Spectra.....	28
9.	Block Diagram of Complete Two-Carrier Amplitude Modulation System as Used in the Experiment.....	33
10.	Photograph of Transmitter Units, Associated Meters, and Power Supplies.....	46
11.	Photograph of Receiver Units, Bench Power Supply Oscilloscope, and Monitoring Speaker.....	47
12.	Schematic Diagram of a Transmitter Unit, Including Antenna Network.....	48
13.	Schematic Diagram of a Receiver Unit, Including Transformer Used in Combininb Signals Differentially....	49

LIST OF TABLES

TABLE	PAGE
I. Observed Cancellation of Received Pulses.....	41
II. Comparison of Received Signals.....	43
III. Components of the Transmitters.....	50
IV. Components of the Receivers.....	51

A SYSTEM OF NOISE REDUCTION EMPLOYING TWO AMPLITUDE MODULATED WAVES

SUMMARY

A system of amplitude modulation is presented which is shown, for a given received signal intensity, to have a greater signal-to-noise ratio than the conventional amplitude modulation system used in commercial radio broadcasting. The system described employs two carrier waves differing in frequency and modulated in phase opposition to each other. The two signals are received in individual receivers, and the resultants of modulation frequency are combined differentially to restore the original signal. It is through the differential connection of receiver outputs that a reduction of impulse noise over that of the output of either receiver is effected.

Two methods (designated as Case 1 and Case 2) of amplitude modulating the carrier waves are described. In Case 1, the individual carrier waves are identical with those of the conventional amplitude modulation system. In Case 2, the amplitude of the carrier frequency component of the transmitted wave varies in direct proportion to the amplitude of the modulating wave. It is shown that for equal total carrier powers a substantial increase in side-band power over that of the conventional system is realized in the method of Case 2.

An experimental system operating over a short distance and employing low transmitted power was used to demonstrate the properties of this system. A noise reduction of the order of magnitude of that predicted was observed.

INTRODUCTION

In all kinds of radio communication systems, noise has always been a topic of great concern. Early wireless systems were not a dependable mode of communication in many locations because of atmospheric interference. Certainly little enjoyment can be derived from listening to a radio program if considerable noise is present.

Radio-telegraph systems can tolerate more noise for satisfactory and readable copy than radio-telephone systems can tolerate for understandable voice transmission. Lifelike reproduction of speech and music requires noise reduction to the point at which it is scarcely more than barely perceptible. High quality television and facsimile reproducing systems are even more exacting as to freedom from noise. The amount of interference which can be tolerated in a radio communication system depends, therefore, upon the nature of the intelligence to be conveyed and upon the subsequent fidelity required.

Noise, regardless of its source, is not the only obstacle to perfection in radio transmission, but in many instances it presents the most difficult problem. There are three places in any system of wireless communication where noise may be introduced: transmitter, transmission medium, and receiver. Usually, reduction to a more than satisfactory degree is readily realizable for noise originating within the transmitter. Atmospheric disturbances in the low- and high-frequency bands and thermal noise in the ultra-high-frequency bands and microwave bands present the most serious noise reduction problems in present systems of radio communication.

The use of increased transmitted power aids effectively in over-

riding atmospherics in all systems; but this expedient has its own limitations. In the case of large transmitters, frequency allocation could become difficult if more than one station were assigned a given frequency unless, however, the power of all stations be increased proportionately. The present trend of the art is toward obtaining a satisfactory ratio of signal intensity to noise level over the desired transmission paths while employing reasonable transmitted powers.

A few of the many means employed in specific systems to gain an effective reduction of noise, other than the use of large powers, are: use of limiters, alteration of receiver bandwidth, and pre-emphasis of certain frequencies within the spectrum occupied by the transmitted intelligence. These methods are largely auxiliary to existing systems of communication and may or may not impair the fidelity of reproduction in the systems in which they are employed.

Transmission at frequencies several times those of the standard American broadcast band is often advantageous in reducing atmospheric noise in all systems of wireless communication. However, long distance communication may become impossible at frequencies above about thirty megacycles per second. Severe fading often plagues long distance short wave communications.

Amplitude modulation was the first system of modulating a carrier wave which gained wide usage. This system of modulation was employed exclusively in established systems of communications until a few years ago. In late years, however, attention has turned to the newer systems of modulation: frequency modulation, pulse-time modulation, and pulse-width modulation. Each of these methods of modulation has characteristic

advantages and disadvantages as compared to the amplitude modulation method. Among the advantages of each is a signal-to-noise ratio greater than that which is possible in the amplitude modulation system for a given power and center frequency. This advantage is due largely to the relative insensitivity of each of these systems to signal amplitude variations at the input of the receiver.

Frequency Modulation. It was believed for many years that a system of frequency modulation did not have an advantage as to noise reduction over the widely used amplitude modulation system. Early experimenters failed to observe a reduction of noise in their frequency modulation systems because the receivers which they used were sensitive to amplitude variations as well as to frequency variations. Successful reduction of atmospheric noises within a system employing frequency modulation depends principally upon making the receiver insensitive (for all practical purposes) to amplitude variations of the incoming signal.

It was not until Armstrong's demonstration¹ of his system of frequency modulation that its property of inherently low noise was realized. His success hinged upon the use of amplitude limiters or saturated amplifier stages preceeding the detector.

Frequency modulation is now widely used in broadcast and voice communications. Modulation in this system is accomplished by changing the carrier frequency by an amount proportional to the amplitude of the

¹Edwin H. Armstrong, "A Method of Reducing Disturbances in Radio Signaling by a System of Frequency Modulation," Proc. I.R.E., 24:689, May, 1936.

intelligence to be transmitted. It is obvious that the transmitted radio-frequency current will vary in frequency above and below that of the center or "resting" frequency. The resultant signal-to-noise ratio in a system of frequency modulation is affected favorably as the magnitude of average frequency swings is increased.

A mathematical analysis of the frequency-modulated wave may be made for sinusoidal modulation by means of Bessel Functions. Such an analysis reveals an infinite series of frequencies separate from the center frequency (which may or may not be present) at intervals of the modulating frequency and extending above and below the center frequency. The series converges more rapidly as the ratio of frequency deviation to modulating frequency (often referred to as the "modulation index") becomes smaller. In practical systems employing frequency modulation the maximum allowable frequency deviation is fixed, and the highest permissible modulating frequency is such that the series will converge rapidly at frequencies lying only slightly outside those corresponding to the maximum allowable frequency deviation.

In designating frequency modulation channels, center frequencies are spaced at intervals somewhat greater than twice the maximum allowable frequency deviation from the center frequency. The width of each channel in the present American frequency modulation broadcast band is 200 kilocycles, and the maximum allowable frequency excursion permitted in the modulation process is ± 75 kilocycles per second. In the bands designated for voice communications only, the channel widths are often about one-sixth as broad.

The Pulse-Time Modulation System. The transmitted wave in a system employing the pulse-time method of modulation consists of a constant frequency carrier wave which is pulsed at a rate that must be at least twice the highest frequency component contained in the modulating wave. The time interval between carrier pulses is usually large compared to the duration of the pulses. Modulation is accomplished by delaying and advancing the time position of these pulses in accordance with the instantaneous amplitude of the modulating wave. The time delay and advance of the pulses will vary retrogressively and progressively in cycles whose frequencies are dependent upon the frequencies present in the modulating wave. In an ideal system a converging infinite series of side frequencies would be produced by pulsing the carrier wave and by modulating it in the manner described.

In a practical system the bandwidth is primarily determined by the build-up time of the pulses and not by the number of pulses used. The bandwidth required is of the order of several megacycles and naturally demands that the carrier frequency be in the very high or super-high frequency regions.

Pulse-Width Modulation System. The pulse-width modulation system is similar to the pulse-time modulation system in that the constant frequency carrier wave is pulsed, and in its bandwidth requirements. Modulation is accomplished by varying the width of the pulses in accordance with the instantaneous amplitude of the modulating wave. The frequency of these cyclical pulse-width variations is that of the modulating wave.

The Amplitude Modulation System. In the amplitude modulation system a carrier wave of constant radio frequency is employed and its amplitude is varied in accordance with that of the modulating wave. Modulation will cause a pair of side frequencies to be introduced, one above and one below the carrier frequency, with a difference from the carrier frequency equal to the modulating frequency. The frequency spectrum occupied by a system of this type is fixed by the modulating frequency and is independent of the amplitude of modulation. The bandwidth required for a system of this type is therefore numerically equal to twice the highest modulating frequency. Each channel has a width of ten kilocycles in the American broadcast band.

THEORY OF THE TWO-CARRIER AMPLITUDE MODULATION SYSTEM

The Two-Carrier System of Amplitude Modulation. The writer has devised a system of amplitude modulation, believed to be novel and of at least academic importance, in which two carrier waves differing in frequency and modulated in phase opposition are employed. The two signals are received and demodulated in individual receivers, and the resultants of modulation frequency are combined differentially. Noise reduction in the two-carrier amplitude modulation system is effected through a partial cancellation of noise by virtue of a balance in receiver outputs.

It is assumed that each transmitter will deliver to its antenna (or to the common antenna if both transmitters supply energy to the same radiating system) a radio-frequency current which is proportional to the instantaneous value of the modulating (or signal) wave. Thus, it is assumed that perfect linearity of the transmitter modulation characteristic exists. The signal current is delivered in opposite phase to the two transmitters. Therefore, when the radio-frequency current delivered by transmitter A is increasing, the current delivered by transmitter B is decreasing at the same rate, and vice versa.

The theory of the two-carrier system of amplitude modulation will be developed for a single-frequency sinusoidal modulating wave. A periodic modulating wave of a different form may be resolved into a Fourier Series of sinusoidal components. The response of the system to such a wave may be studied by treating each component in a manner similar to that used for the single-frequency modulating wave. The principle of superposition may then be employed in evaluating the final

results.

Two methods of amplitude modulating the carrier wave in the two-carrier system are described as follows:

Case 1. The radio-frequency wave emanating from each transmitter varies in amplitude about a fixed magnitude in exact accordance with the modulating wave. The modulation process in each of the transmitters is identical to that in a transmitter of the conventional system of amplitude modulation. The modulation process introduces in the output of each transmitter two additional sinusoidal components, one higher and one lower in frequency than the carrier frequency, whose coefficients vary in proportion to the steady state amplitude of the modulating wave. Each component, or side-band, differs from the carrier frequency by an amount equal to the frequency of the modulating wave. Figure 1 shows the modulating wave, a wave of radio-frequency, and the modulated radio-frequency waves of Case 1.

Case 2. Modulation of each carrier wave in this case is accomplished in much the same way as in Case 1. Now, however, the radio-frequency output is zero in the absence of a modulating signal. Each of the two radio-frequency output waves is proportional in amplitude to the instantaneous amplitude of the signal current received by its respective transmitter, but the individual transmitter delivers radio-frequency current to the antenna only when the signal current received by it is positive. Therefore, each transmitter delivers current to the antenna for a period less than the time of a full cycle of the modulating wave. In the case of a sinusoidal modulating wave, which is used in making the following analysis, each transmitter delivers current to the

antenna during alternate halves of the modulation cycle, or half of the total time. With sinusoidal modulation, the envelopes of the radio-frequency oscillations will appear as half-sines located above and below the horizontal axis (time axis) and spaced at intervals of one-half cycle of the modulating wave as shown in Figure 2. This method of radio transmission is analogous to the widely used class "B" audio amplifier.

It becomes apparant that the side-bands associated with the carrier in the system of Case 2 are more numerous than those in the system of Case 1. The following analyses will reveal the extent of side-band generation and the minimum transmitted bandwidth necessary for satisfactory reproduction of the intelligence. The method of Case 2 will be referred to from time to time in this thesis as the "half-envelope method."

Frequency and Power Relations. The equation of the transmitted wave emanating from one transmitter of Case 1 is developed in Equations (1), (2), and (3). Equation (3) is identical with the equation of the transmitted wave in the conventional system of amplitude modulation and serves to illustrate the frequency and amplitude of the side-band currents with respect to the carrier current. Equation (4) presents mathematically the relationship of the total side-band power contained by one transmitted wave to the power contained by one carrier wave. Inspection of Equation (4) reveals this ratio to be a function of the magnitude of the signal current, with a maximum value equal to $1/2$.

The modulating wave of Case 2 takes the form of half-sines spaced by intervals equal to their duration and located on one side of the axis.

Modulation is accomplished by varying the amplitude of the radio-frequency wave in accordance with the intelligence being transmitted. This is stated mathematically by Equation (5). Derivation of the equation of the modulated wave is readily performed by expanding the modulating signal into a Fourier Series, Equation (6), and substituting it into Equation (5). From this substitution, Equation (7) is obtained. Figure 2 depicts the signal wave and the axes chosen for resolving it into the series. In the general case of non-sinusoidal modulation, the modulation envelope can also be expressed as either a Fourier Series or a Fourier Integral, according to whether the modulation envelope is a periodic or non-periodic function of time.

Inspection of Equation (8) reveals that the series converges rapidly. The line-spectrum diagram of Figure 4a serves in illustrating this convergence. Power contained in a wave is proportional to the sum of the squares of the crest values of all of its sinusoidal component coefficients. Several coefficients from Equation (8) have been treated in this manner and plotted in Figure 4b. It is evident that a relatively large amount of power is contained in the side frequencies when this transmission system is modulated sinusoidally; and that the majority of it lies in the first two pairs of side frequencies. Total power contained by the modulated wave is proportional to the mean-square value of the signal wave ($I^2/4$); and Equation (8) reveals that the carrier power is proportional to (I^2/π^2) . Logically, the power contained by all of the side-bands will be the difference in total power and carrier power. The ratio of the side-band power to carrier power is computed in Equation (9) and remains constant for any amplitude of the modulating wave. This ratio is found to be 1.4674... .

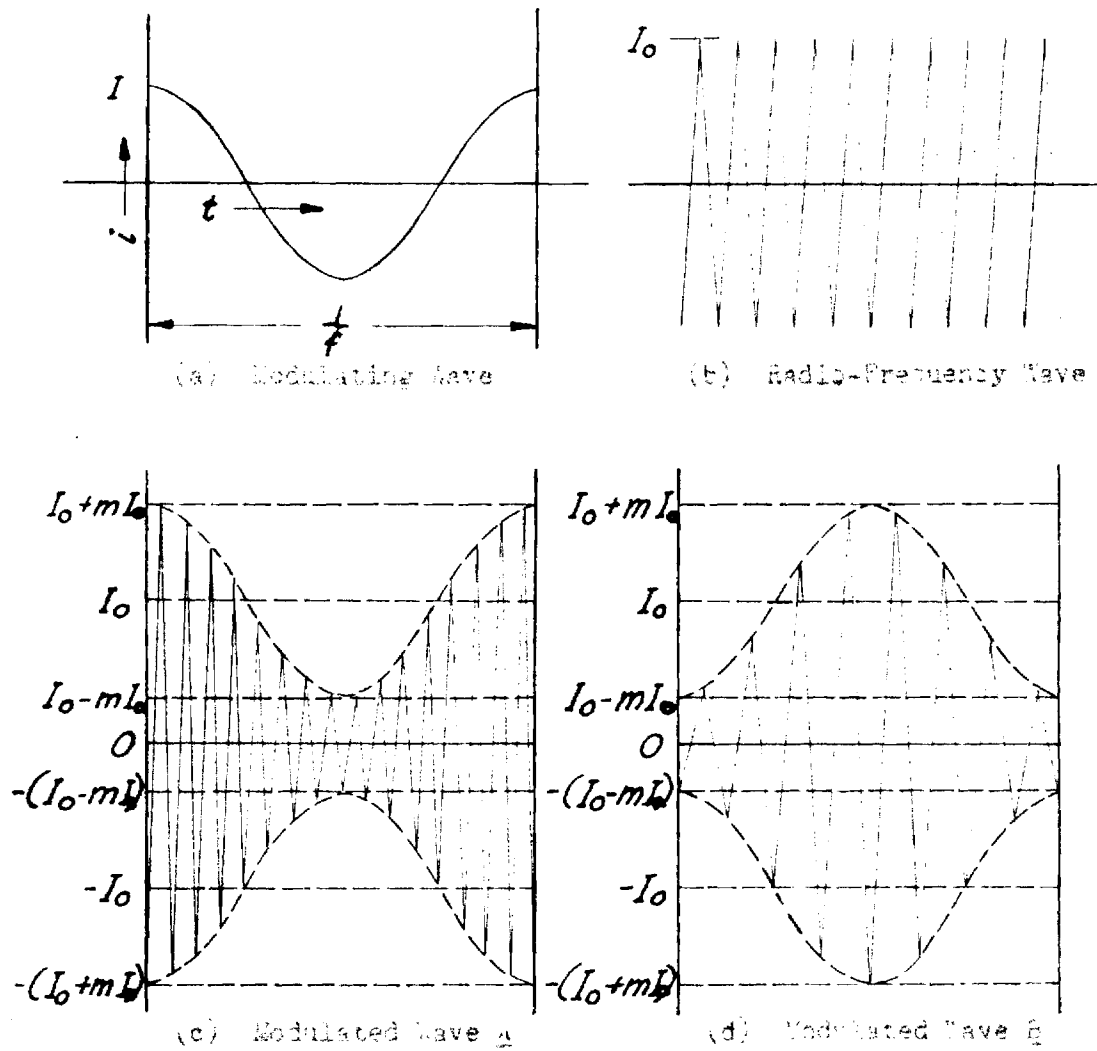


Figure 1.

$$m = \frac{I}{I_0} \quad (1)$$

$$i = m I_0 \cos \omega t \quad (2)$$

$$i_{RF} = (I_0 + m I_0 \cos \omega t) \cos \Omega t = I_0 \left[\cos \Omega t + \frac{m}{2} \cos(\Omega t + \omega t) + \frac{m}{2} \cos(\Omega t - \omega t) \right] \quad (3)$$

$$\frac{\text{Total Side Band Power}}{\text{Carrier Power}} = \frac{m^2}{2} \quad (4)$$

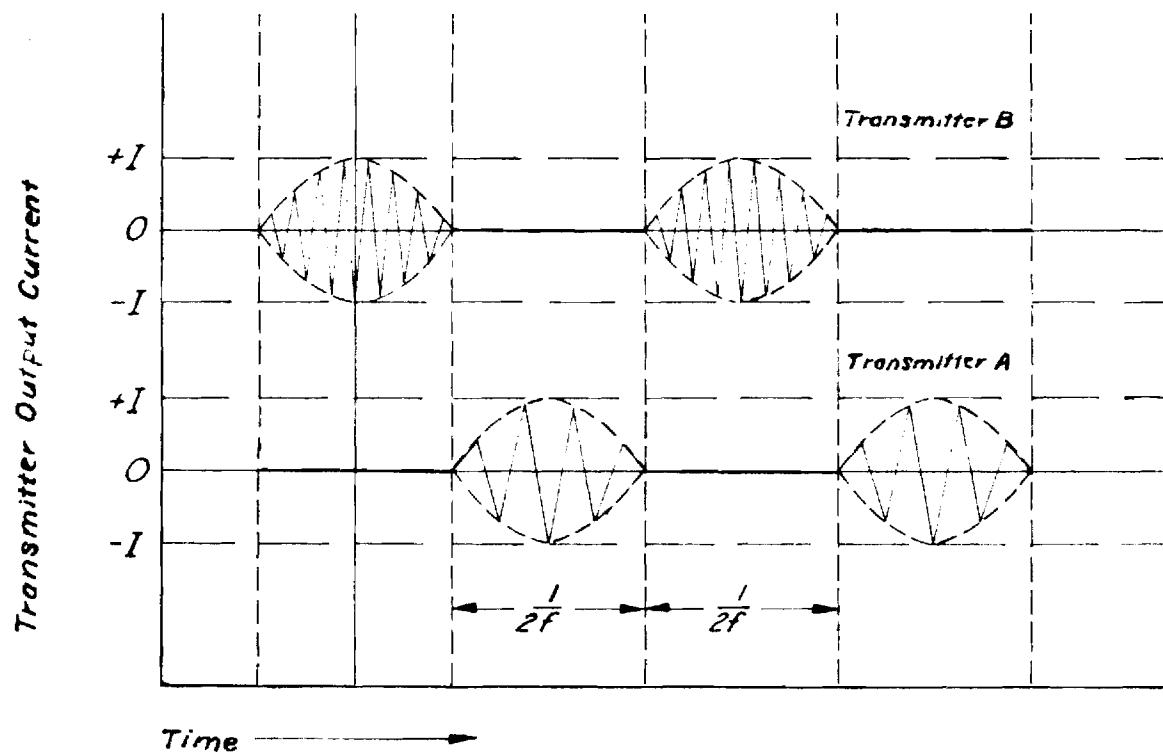


Figure 2. Modulated Waves Emanating from Each Transmitter.

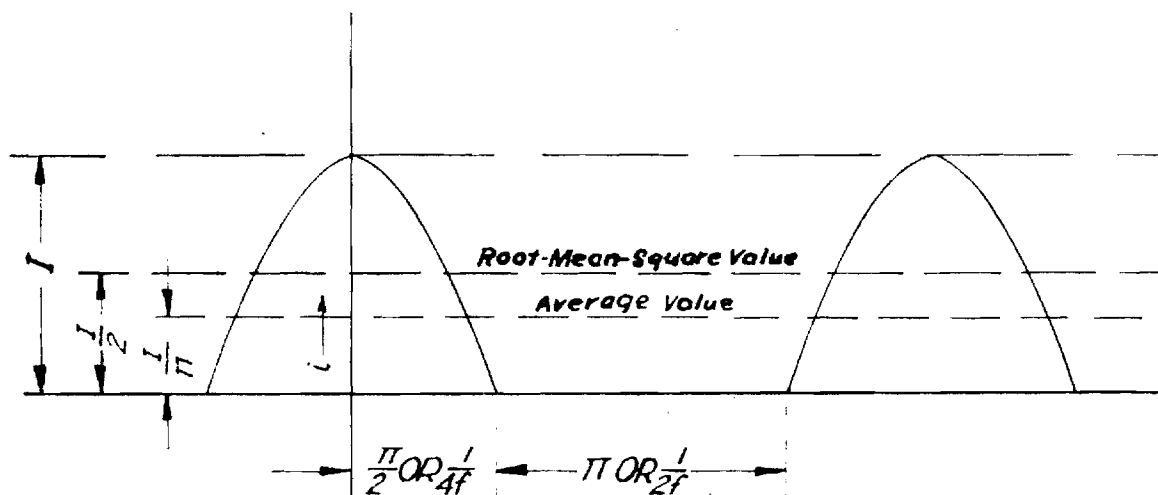


Figure 3. Modulating Wave with Axes for Resolution and Lines Indicating Crest, Root-Mean-Square, and Average Values of the Wave.

$$i_{rf} = i \cos \Omega t \quad (5)$$

$$i = \frac{I}{\pi} \left[1 + \frac{\pi}{2} \cos \omega t + \frac{2}{3} \cos 2\omega t - \frac{2}{15} \cos 4\omega t + \frac{2}{35} \cos 6\omega t + \dots (-1)^{\frac{n}{2}+1} \frac{2}{n^2-1} \cos n\omega t \right] \quad (n \text{ even}) \quad (6)$$

$$i_{rf} = \left\{ \cos \Omega t \right\} \left\{ \frac{I}{\pi} \left[1 + \frac{\pi}{2} \cos \omega t + \frac{2}{3} \cos 2\omega t - \frac{2}{15} \cos 4\omega t + \frac{2}{35} \cos 6\omega t + \dots (-1)^{\frac{n}{2}+1} \frac{2}{n^2-1} \cos n\omega t \right] \right\} \quad (n \text{ even}) \quad (7)$$

$$i_{rf} = \frac{I}{\pi} \left\{ \cos \omega t + \frac{\pi}{4} [\cos(\Omega t + \omega t) + \cos(\Omega t - \omega t)] + \frac{1}{3} [\cos(\Omega t + 2\omega t) + \cos(\Omega t - 2\omega t)] - \frac{1}{15} [\cos(\Omega t + 4\omega t) + \cos(\Omega t - 4\omega t)] + \dots (-1)^{\frac{n}{2}+1} \frac{1}{n^2-1} [\cos(\Omega t + n\omega t) + \cos(\Omega t - n\omega t)] \right\} \quad (8)$$

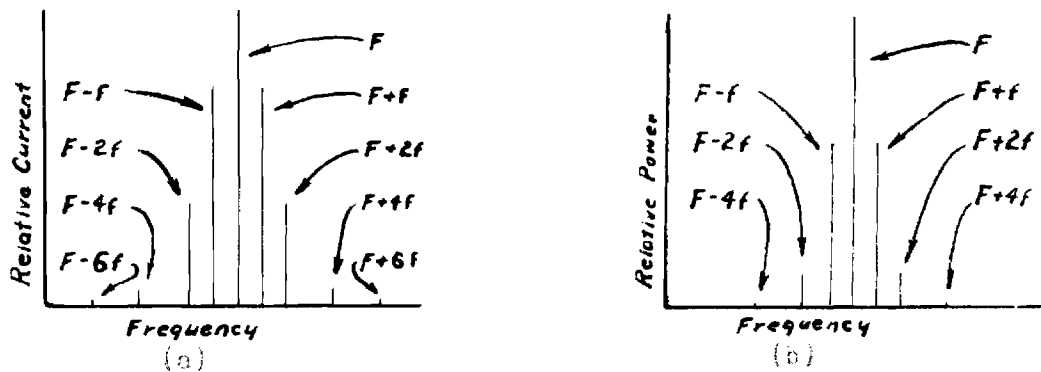


Figure 4. Carrier and Side-Band Currents and Powers Represented as the Lines of a Frequency Spectrum.

$$\frac{\text{Side Band Power}}{\text{Carrier Power}} = \frac{\frac{I^2}{4} - \frac{I^2}{\pi^2}}{\frac{I^2}{\pi^2}} = \frac{\pi^2}{4} - 1 = 1.4674 \dots \quad (9)$$

Noise Reducing Properties. Noise is defined as interference whose energy is distributed over a wide band of frequencies.² It is considered to be made up of sharp pulses, and the frequency with which the pulses occur determines the character of the noise. If the pulses are relatively infrequent and clearly separated, the noise is said to be impulsive. If, on the other hand, the pulses follow each other so rapidly that they overlap and are not clearly distinguishable, then the noise is said to be random or smooth.³ The effect of each of these types of noise currents upon the system will be studied individually. For this analysis it will be assumed that the two differentially connected receivers have circuits of identical selectivity and, hence, have identical time constants at the different frequencies to which the receivers are of necessity tuned. The receivers are also assumed to have equal gains, and, therefore, equal signal inputs for the balance of output signals to be attained. Lastly, it is imperative that the overall phase shift of each receiver be the same. If two identical superheterodyne receivers using equal intermediate frequencies are employed, these conditions are almost exactly fulfilled. This is true because the intermediate frequency amplifier in a superheterodyne is the major factor in determining the receiver sensitivity and selectivity.

No reduction of random noise is possible through an inherent property of the two-carrier amplitude modulation system. If the sharp,

²"Standards on Radio Wave Propagation. Definitions," Proc. I.R.E., Supplement to Vol. 30, Part III:4, 1942.

³Keith Henney, Radio Engineering Handbook (McGraw-Hill Book Co., 1941), p. 531.

overlapping pulses of which random noise is composed are resolved into a frequency spectrum, there results an infinitude of sinusoidal components of random phase whose frequencies possess no particular harmonic relationship. Each receiver, although tuned to a different carrier frequency, will intercept an equal portion of an infinite spectrum of noise frequencies in addition to its intended signal frequencies. No cancellation, or partial cancellation, of noise currents can occur by means of the differential connection of receiver outputs, for such would require that the noise currents lying in the pass band of the two receivers be related in magnitude, phase, and frequency. The noise currents present in the output of a given receiver will thereby always add in some way to those in the output of the other receiver, regardless of the connection employed (differential or accumulative). The mean square value of random noise current is proportional to receiver bandwidth.

Random Noise in Case 1. A transmission system employing the two-carrier modulation method of Case 1 will require a total receiver bandwidth of four times that of the highest modulating frequency to be transmitted. It follows that the mean square value of random noise current intercepted by the two receivers will be twice that intercepted by a receiver employed in the conventional single-carrier amplitude modulation system. If the power emanating from each transmitter of the two-carrier system is one-half that emanating from a single transmitter of the conventional system, the amplitude of the transmitted wave will be $1/\sqrt{2}$ that of the amplitude of the wave of the conventional system. At the receiver output, the two waves will combine on a voltage basis into a resultant wave having an amplitude numerically equal to $\sqrt{2}$ times

the amplitude of the wave appearing at the output of the conventional system receiver. The power of the reproduced modulating wave appearing at the output of the differentially connected receivers will therefore be twice that appearing at the output of the conventional receiver. Because both noise and signal powers are doubled, the signal-to-random noise ratio of signal output from the receivers of the two-carrier amplitude modulation system will be equal to that of a single carrier (or conventional) amplitude modulated system employing a transmitted power equal to the total transmitted power of both transmitters and a receiver bandwidth equal to that of one receiver in the two-carrier system.

Random Noise in Case 2. The relationship of receiver bandwidth to intercepted random noise currents was discussed in the analysis of the effect of random noise in the system of Case 1 of the two-carrier system. The total bandwidth required of the receiver was definitely fixed by the fact that only two side-bands are produced by the modulating wave. The minimum bandwidth required for satisfactory transmission of the intelligence in Case 2 of the two-carrier system must first be determined before a theoretical study of the effect of random noise currents upon this system can be carried through. Obviously, when a limited band of side frequencies is transmitted or received, distortion of the intelligence to be transmitted will occur.

The transmitted radio-frequency current may be represented graphically by allowing a system of rotating vectors to describe the carrier current and the side currents to be considered. The vectors will rotate at angular velocities proportional to the frequencies of the currents

represented by them, and their lengths will be proportional to the amplitudes of their respective currents. For example, such a representation of the completely modulated radio-frequency wave emitted from a transmitter of the conventional amplitude modulation system, or a single transmitter of Case 1, will show a sine curve whose crest value is undergoing cyclical variations at a frequency equal to the frequency difference between the carrier frequency and each side frequency. The two vectors representing the side-bands for complete modulation will each have a length one-half that of the carrier vector and at some instant of time, both will act in concert with the carrier vector to give the maximum value that the crest value of the radio-frequency wave may have. Likewise, at some other instant of time, both side frequency vectors will again coincide but will have a direction opposite to that of the carrier vector, thus giving a combined resultant of the three equal to zero.

Investigation of the minimum bandwidth requirement is facilitated by considering the transmitted wave envelope as being described by a system of vectors representing the carrier frequency and associated side-bands to be considered. The carrier vector will be of unit length and fixed; the lower frequency side-bands will be represented by vectors of appropriate lengths relative to that of the carrier vector and rotating in a clockwise direction. The upper side currents will be represented by vectors similar to those of the lower side currents, but rotating in a counter-clockwise direction. The velocity of the rotating vectors will be proportional to the frequency difference between their associated frequencies and the carrier frequency. Because the carrier vector was fixed, a mirror image of the curve described must be

constructed about the horizontal axis so that the complete radio-frequency wave envelope will be described.

Demodulation will remove the portion of the wave envelope lying in the region on one side of the horizontal axis. Figure 5a shows the detected wave envelope plotted for the carrier and first pair of side currents. The waves from both channels have been placed in their proper relationships on the same axes and the resultant of both is included. It is obvious that distortion, excessive for many applications, will result. Figure 5b depicts a set of curves plotted in a similar manner to those of Figure 5a, but includes the first two pairs of side currents.

The harmonic content of the resultant wave of Figure 5b has been computed from twelve ordinate (evenly spaced) values.⁴ The fundamental frequency current has a crest value of 3.031; the third harmonic current is 5.21% of the fundamental; and the fifth harmonic is 1.55% of the fundamental, thus giving a total harmonic content of 5.43% for harmonic currents up to and including the fifth. No even order harmonics of the signal frequency are present in the resultant wave because of its symmetry about the horizontal axis. It is apparent that inclusion of the first two pairs of side currents will produce a resultant wave at the output of the differential receiver satisfactory for most voice and many music reproduction purposes.

It can be deduced from an inspection of Figure 3 and Equation (7) that the ratio of resultant signal wave crest to carrier current crest is π if all side-bands are transmitted and received without attenuation

⁴Ibid., p. 21.

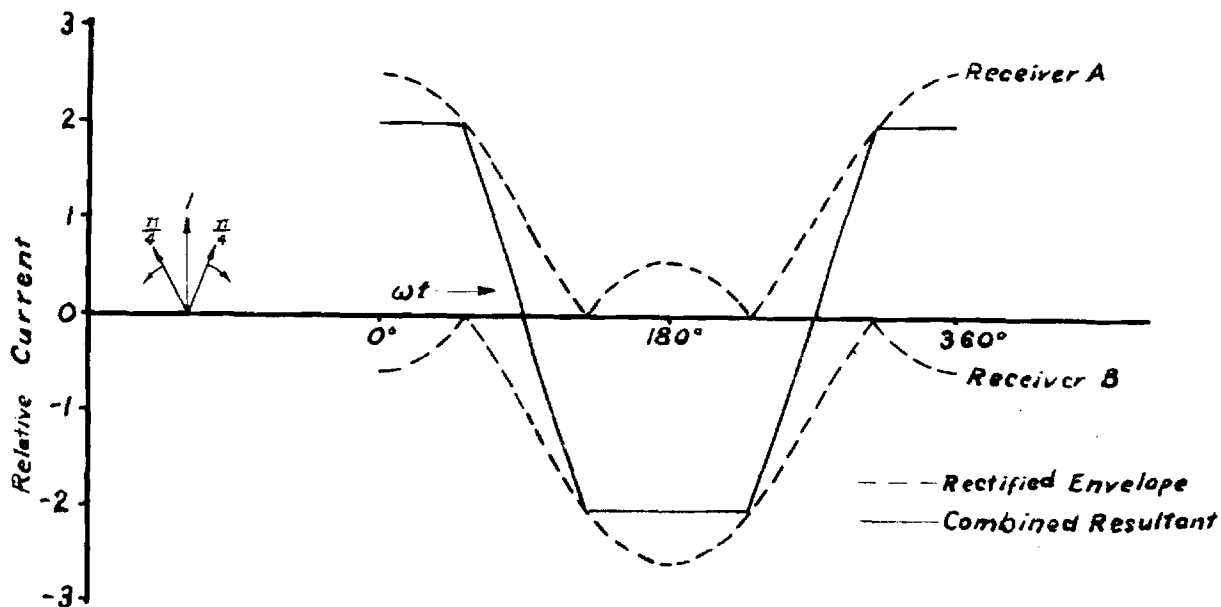


Figure 5a. Carrier and Side-Band Vectors, and Rectified Envelopes and Resultant produced by the Carrier and First Pair of Side-Bands.

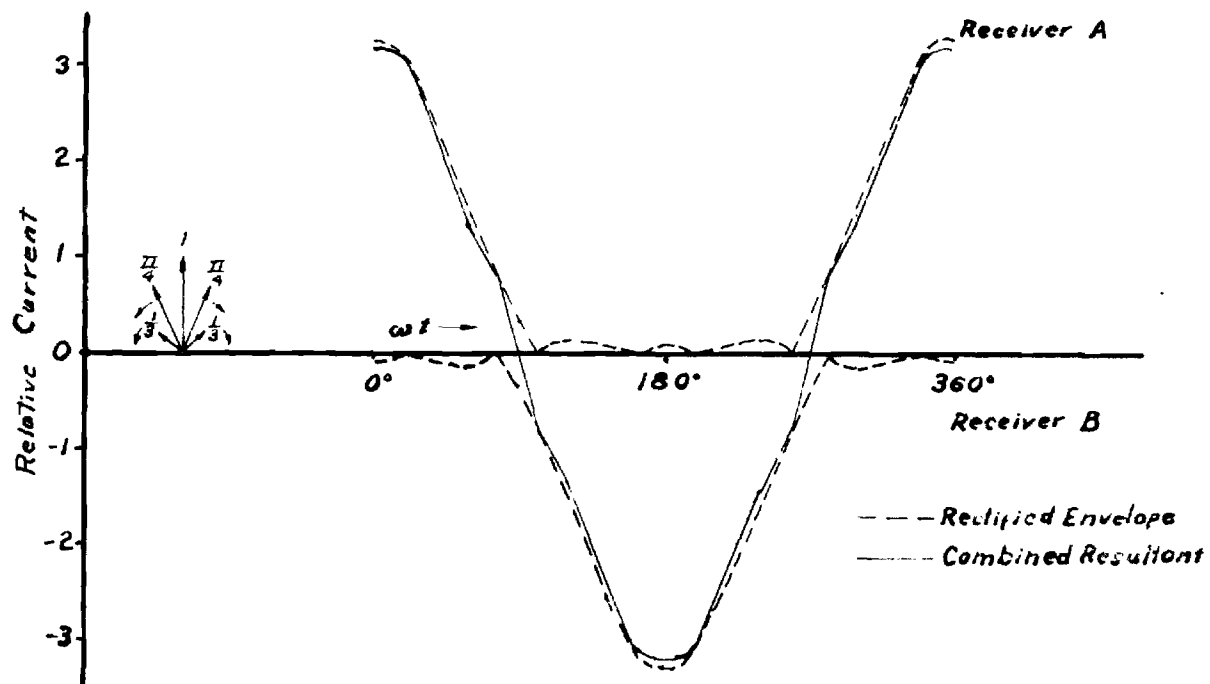


Figure 5b. Carrier and Side-Band Vectors, and Rectified Envelopes and Resultant produced by Carrier and First Pair of Side-Bands.

or phase distortion. From the analysis made to determine the harmonic content of the wave reproduced by transmission of the two carriers (crest values = unity) and first two pairs of unattenuated side currents associated with each, it is found that the crest of the fundamental component of the signal wave is reduced to 3.031. The bandwidth for each channel in the system of Case 2 will be taken as being equal numerically in cycles to four times the highest modulation frequency to be transmitted. Side-band limiting would usually be employed at the transmitter so as to conserve band space. If the crest value of the carrier current of each channel is maintained at unity, and if the modulation frequency approaches zero from its highest value (as determined by the system bandwidth), then the crest value of the resultant wave will approach π and the harmonic distortion will approach zero. It follows that with a fixed bandwidth and fixed carrier current value, the signal-to-random noise ratio will increase slightly as the modulation frequency is reduced; but the improvement will occur only as additional side frequencies are included.

The investigation of the signal-to-random noise ratio in the two-carrier system of Case 2 will be pursued using the conventional single carrier amplitude modulation system for comparison. It is assumed that the power of the carrier frequency in each channel of the system is one-half that of the carrier frequency power in the conventional system, giving equal total transmitted carrier powers in both systems. The modulation frequency is assumed to be much lower than one-fourth the bandwidth of each channel in Case 2, so that the resultant signal wave crest can be considered as being equal numerically to π times the crest value of

either of the two carrier waves. Hence, the resultant signal wave may be considered as being an undistorted reproduction of the modulating wave.

The carrier of the conventional system may be considered as capable of being modulated to any degree not greater than 100%. The modulating current for the system of Case 2 is taken as being of such a value that the total carrier power of the system is equal to the carrier power of the conventional system. In the conventional system the carrier power is fixed if modulation does not exceed 100%, but in the two-carrier system of Case 2 the carrier power is a function of the modulating current. Thus, it follows that the signal-to-random noise ratio of Case 2 as compared to that of the conventional system will become a function of the percentage modulation, m , of the conventional system when equality of carrier powers is maintained. The carrier current amplitude of each channel will be $1/\sqrt{2}$ that of the conventional system; the transmitted side-bands will be reduced proportionally giving a resultant wave of $\pi/2$ whose power is $\left[\frac{\pi}{\sqrt{2} m} \right]^2$ that of the power in the reproduced signal in the conventional system. The percentage modulation, m , may have any value between 0 and 1, and has previously been defined as the ratio of signal current to carrier current. The total bandwidth required in Case 2 will be twice that of Case 1, or four times that of the conventional system, resulting in a random noise power interception of four times the random noise intercepted by the conventional system. The signal-to-random noise ratio of Case 2 then has a value of $\frac{1}{8} \left[\frac{\pi}{m} \right]^2$ that of the signal-to-random noise ratio of the conventional system. Expressing the signal-to-random noise ratio of the system of Case 2 over that of the conventional system in decibels, we have

$$\text{Decibels} = 10 \log_{10} \frac{1}{8} \left[\frac{\pi}{m} \right]^2 . \quad (10)$$

The signal-to-random noise ratio gain in the half-envelope system over that of the conventional system is only 0.9 decibel if m is maintained at its maximum value. In telephonic speech, the percentage modulation may be assumed to have an average value of 20% ($m = 0.2$) of the maximum value over a considerable period of time.⁵ This corresponds to an average signal-to-random noise ratio gain of 14.9 decibels.

It has been pointed out that transmitting the carrier and only the first two pairs of side-bands in each channel will distort the resultant signal wave from the original. A reduction of the fundamental frequency component in the output of the receivers by the factor $\frac{3.031}{\pi}$ will cause the signal-to-random noise ratio gain in Case 2 to be reduced about 0.3 decibel.

The Effect of Impulse Noise on Cases 1 and 2. The pulses of which impulse noise is composed are by definition clearly separated. Much of the noise commonly intercepted by receivers is of the impulsive type. A pulse of energy entering a radio receiver will set the tuned circuits into oscillation at their resonant frequencies by impact excitation. Consequently, static and other irregular electrical impulses will be received regardless of the tuning adjustments of the set.⁶

⁵Sidney T. Fisher, "A New Method of Amplifying with High Efficiency a Carrier Wave Modulated in Amplitude by a Voice Wave," Proc. I.R.E., 34:3P, January, 1946.

⁶R. S. Glasgow, Principles of Radio Engineering (McGraw-Hill Book Co., 1936), p. 112.

It has been assumed that the two receivers in the two-carrier system are identical electrically, but are tuned to different frequencies. It is further assumed that linear peak operated detectors are employed. Shock excitation of the tuned circuits by a pulse will then produce at the output of each detector half-envelopes of damped sinusoidal wave trains. All tuned circuits are assumed to have equal decrements (or time constants). The instantaneous amplitudes of the wave train envelopes will not in general be equal; therefore, complete elimination of the effect of the received pulses will not ordinarily be realized. The analysis for impulsive noise applies equally for Cases 1 and 2 of modulation. It will be pursued along a line of combined mathematical and physical reasoning. The object of this analysis is to show that a partial cancellation of the effect of the received impulses is possible.

It is known that the pulses making up impulse noise are usually separated by an interval of time which is greater than the duration of the pulses and that the time interval is of a random nature. Thus, the pulses composing impulse noise are not in general a periodic function of time. One exception exists in interference generated by electrical machinery rotating at a constant angular velocity. Lastly, the time interval between pulses is very often great enough so that the transient oscillations in the receiver tuned circuits practically vanish before the next impulse occurs. Such is assumed to be the case in making this analysis.

It is impossible to formulate a pulse of a general nature to serve as a basis for a study of the behavior of impulse noise upon the two-carrier system of amplitude modulation. However, a regularly recurring

rectangular wave whose time duration is short compared to its interval and whose repetition frequency is much lower than any frequency lying within the pass band or either receiver should yield results which are typical and significant. Such a wave is assumed in this study.

The rectangular wave is shown in Figure 6. Equation (13) is the Fourier Series of the wave written as a summation. The line spectrum diagram of Figure 7a shows the relative magnitudes of the Fourier coefficients and the manner in which they converge. The relative magnitude of the average (or zero frequency coefficient) component is represented in Figure 7 by the distance from the horizontal axis to the intersection of the curve and the vertical axis. Each of the coefficients is spaced along the abscissa of Figure 7 at intervals equal numerically to the repetition frequency of the pulses. The abscissa of Figure 7a then represents frequency and continues to infinity when the pulses are perfectly rectangular. As the pulse duration is made shorter, the curve of coefficients shown in Figure 7a will cross the horizontal axis (frequency axis) at greater intervals of frequency as shown in Figure 7b. The coefficients of the infinite series converge less rapidly as the pulse duration is shortened, and the magnitudes of the coefficients will be reduced generally. In the limit, where the duration of the pulses approaches zero, all frequency components will have equal coefficients, but each coefficient will approach zero in magnitude.

It is assumed that the differential receivers are tuned so that their pass bands of equal width are approximately adjacent to each other, but do not overlap. All frequencies lying within the pass band of either receiver are by assumption much higher than the repetition frequency of

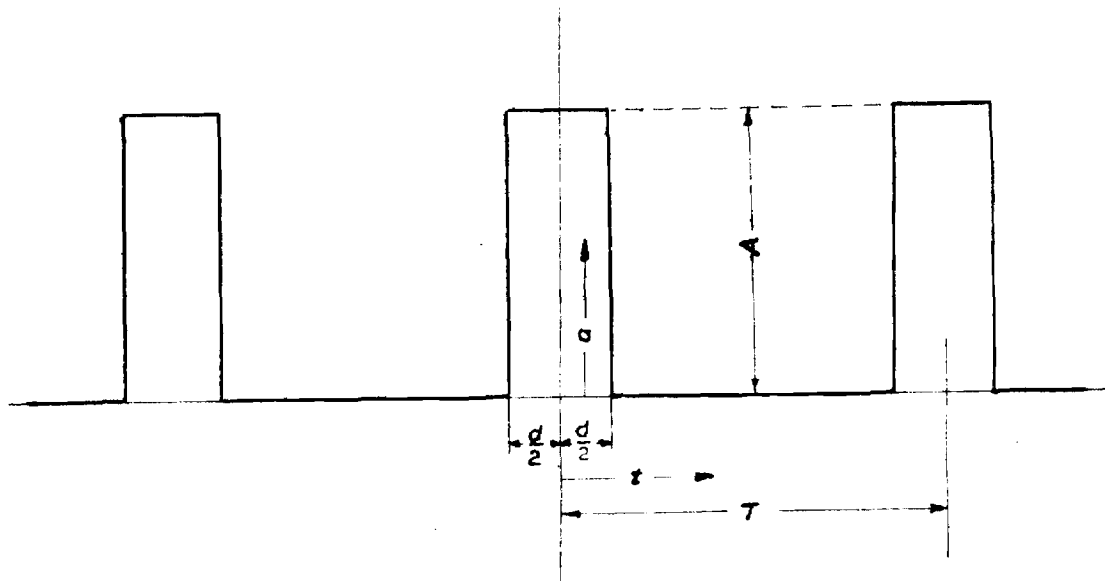


Figure 6. Form of the Rectangular Wave, Axes for Resolution of the Wave into a Fourier Series, and Dimensions.

$$\frac{1}{T} = f_0 \quad (11)$$

$$2\pi f_0 = \omega_0 = \frac{2\pi}{T} \quad (12)$$

$$a = \frac{2Ad}{T} \sum_{n=0}^{\infty} \left(\frac{\sin\left(\frac{n\pi d}{T}\right)}{\frac{n\pi d}{T}} \right) \cos n\omega_0 t \quad (13)$$

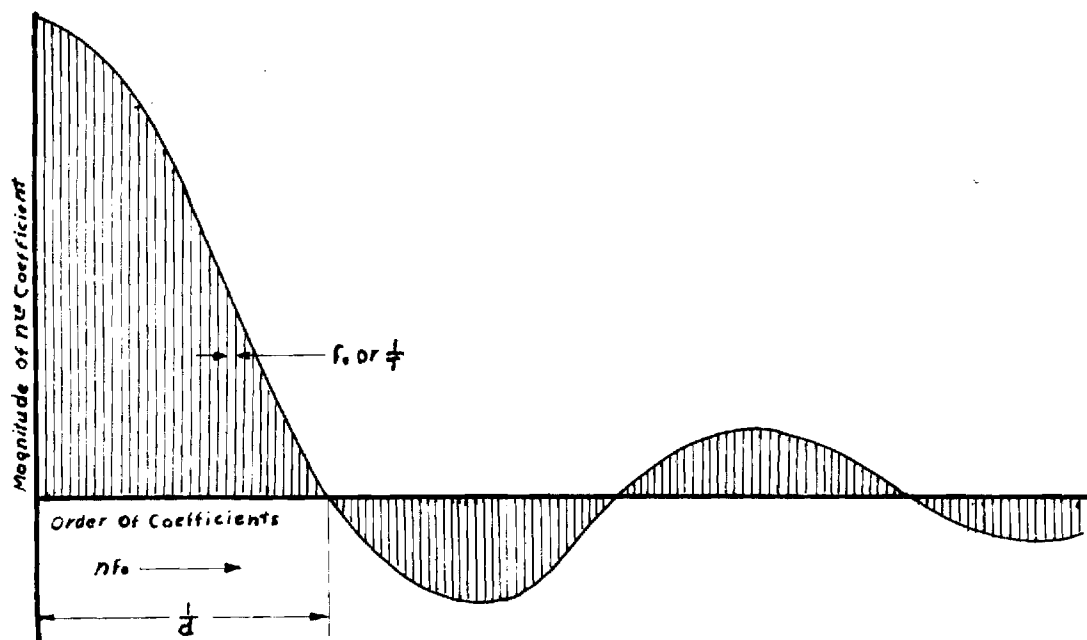


Figure 7a. Manner in Which Wave Coefficients Vary in Magnitude.

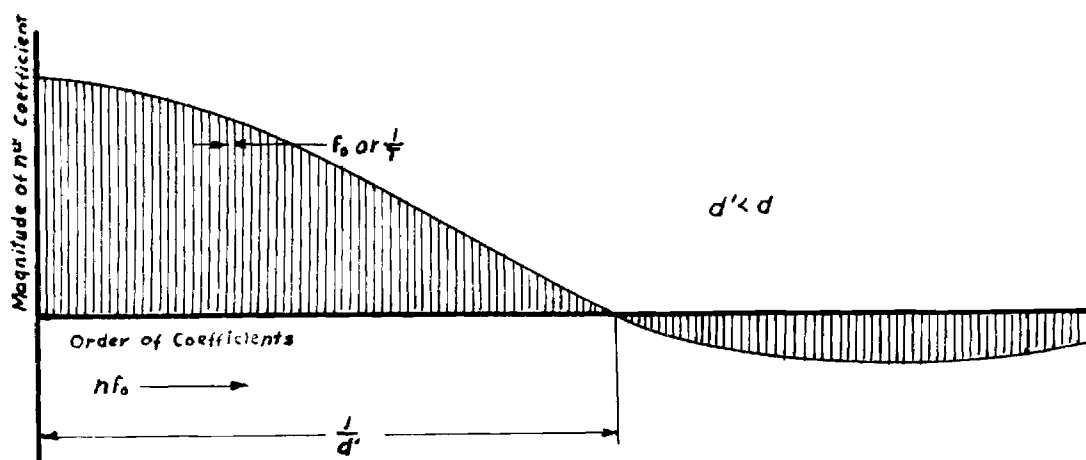


Figure 7b. Manner in Which Wave Coefficients Vary in Magnitude for Shorter Pulse.

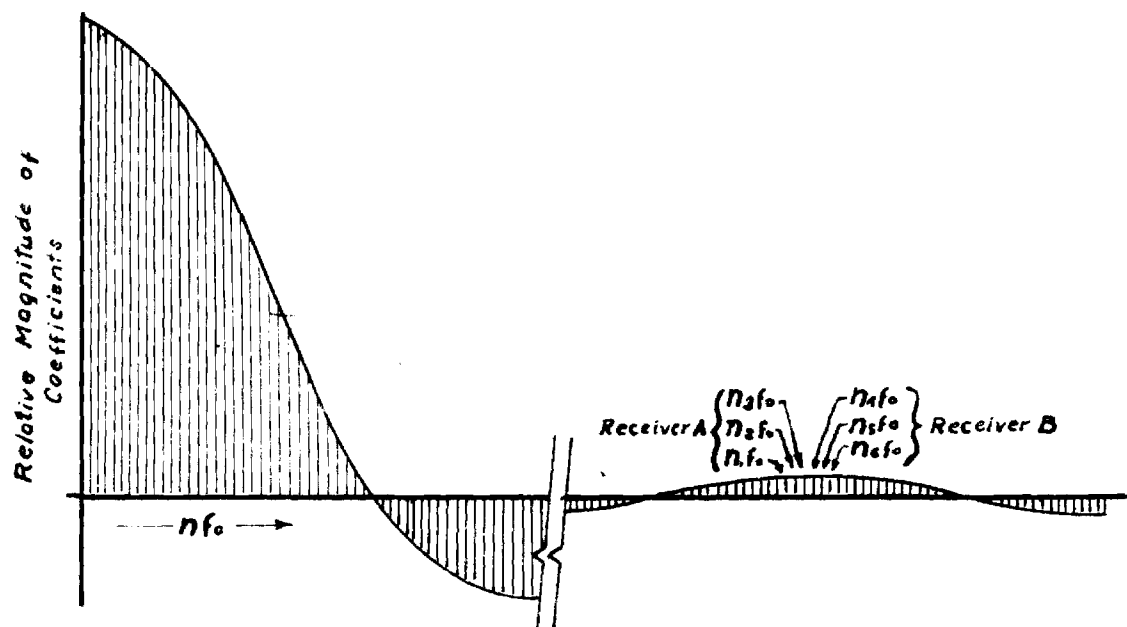


Figure 8a. Spectra of Rectangular Wave Components Intercepted by Each Receiver.

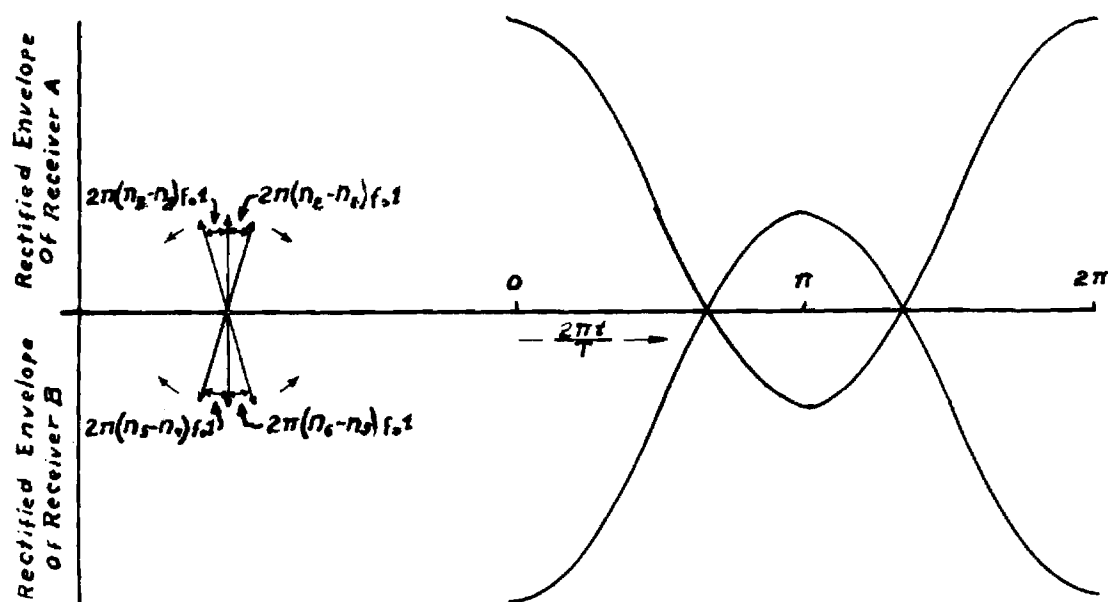


Figure 8b. Rectified Envelopes Produced by Two Spectra of Three Components, All Having Equal Coefficients.

the pulses. The pass band of the receivers is very narrow compared to the mid-frequency of either. The intercepted currents entering each receiver may be represented by a system of rotating vectors, each having length equal to the amplitude of the sinusoidally varying current that it represents and rotating at an angular velocity (in radians per second) of 2π times the frequency of the component represented by it. A set of vectors representing the components of the pulses intercepted by a receiver will, when added vectorially for every instant of time and plotted on time-magnitude coordinates, describe the instantaneous resultant oscillation entering the receiver. If the vector representing the mid-frequency intercepted component be considered as fixed instead of rotating and the other vectors be considered as rotating at velocities proportional to their frequency difference from that of the fixed vector, then one-half of the radio-frequency envelope due to the effect of the pulses will have been described. The complete envelope is described only after a mirror image is constructed on the opposite side of the horizontal axis from that preceeding because the mid-frequency vector was considered fixed. Detection will serve to remove all of the envelope lying on one side of the horizontal axis. The character of the envelope due to the intercepted components of the pulses is dependent only upon the repetition frequency of the pulses (which is assumed constant) and to the values of coefficients of the components.

It has been pointed out that a pulse duration which is short compared to the repetition time will result in a slow convergence of pulse component coefficients. Also, the bandwidths of the receivers have been assumed to be equal and small compared to their center frequencies.

Under such circumstances, if both receivers are tuned to adjacent or nearly adjacent spectra of pulse components at or near a point of zero slope of the coefficients curve (shown in Figure 8a), all the intercepted components will produce substantially identical rectified envelopes appearing at the output of their detectors. Under these conditions a nearly complete cancellation of the effects of the pulses will be realized. To illustrate the principle by which cancellation occurs, six vectors of equal length representing two spectra of pulse component coefficients have been drawn. Four are shown rotated by equal amounts. The rectified envelopes due to each group are plotted. The resultant effect of the pulses will be zero in this instance. This simple example would require that the pulse repetition frequency be greater than one-fourth the bandwidth of each receiver. In a practical case more components would have to be considered, but the general result would be the same.

In general, all intercepted component currents of the pulses will not have equal magnitudes. Also, in general, various components will be shifted in phase by the tuned circuits in the receivers, but every component in a given spectrum will have a corresponding component in the other spectrum (which may or may not be equal to it) whose phase is displaced an equal amount if, as we assumed, the decrements of the tuned circuits are equal.

Interception of pulse components may occur in spectra located near each other at any points on the curve of coefficients, but little reduction of the effects of the pulses will be realized unless corresponding components in the different spectra are of the same order of magnitude. If the spectra of intercepted pulse components are adjacent and are

located symmetrically about a zero point on the curve of coefficients, then minimum cancellation of the effect of the pulses will occur. However, in this case, the magnitudes of the coefficients are relatively small as compared to those lying in regions near the points midway between the zero points. This is to say that the two-carrier system would be relatively insensitive to the incoming pulses over a wide range of pulse durations.

It was stated previously that the pulses of which impulse noise is composed are not in general a recurrent function of time. A finite number of pulses having differing time durations and time spacings can be resolved into a spectrum of frequencies by means of Fourier Integral methods,⁸ to make the study of the effect of impulsive noise upon the system more general. Such a resolution, whether for a single pulse or many pulses not having equal time spacings, would reveal a spectrum of frequencies spaced from each other by differential increments of frequency. Each coefficient would therefore be differential in magnitude for pulses having an energy content less than infinity. The analysis which shows the effect of pulses of random time intervals requires that all pulses be stated as Fourier Integrals. Each integration must be performed between the pass band limits of each receiver for all instants of time for which the analysis is concerned, and the principle of superposition must be applied to evaluate the currents due to the components intercepted by the respective receivers.

⁸T. E. Shea, Transmission Networks and Wave Filters (D. Van Nostrand Company, Inc., 1929), pp. 417-426.

APPARATUS AND EXPERIMENTAL DATA

General Description of the Amplitude Modulation System. An experimental system of amplitude modulation employing two carrier waves differing in frequency and modulated in phase opposition was set up in the laboratory. A short transmission path, approximately forty feet between transmitting and receiving antennas, was used because of limited space and so that a minimum amount of transmitted power could be employed to advantage.

The transmitters were installed in a shielded room, and the radio-frequency output of each fed to a common antenna located outside the room. The source of audio-frequency modulating current and the transmitted signal monitoring system were also located outside the room and were connected with the transmitter through suitable wiring.

The receivers were located on the same floor as the transmitters but were not installed in the shielded room. A short antenna located inside the room was connected to each receiver. Necessary test equipment and a loud speaker, which was used for monitoring purposes, were located on the receiver bench.

Interference for test purposes was introduced externally from the transmitters and receivers, and interference from several sources was observed.

A block diagram depicting all components of this experimental system is shown in Figure 9.

Description of the Transmitters. Two new, modified U. S. Army Signal Corps transmitters, type BC 458-A, with an operating range of frequencies from 5.3 to 7.0 megacycles per second were employed. The

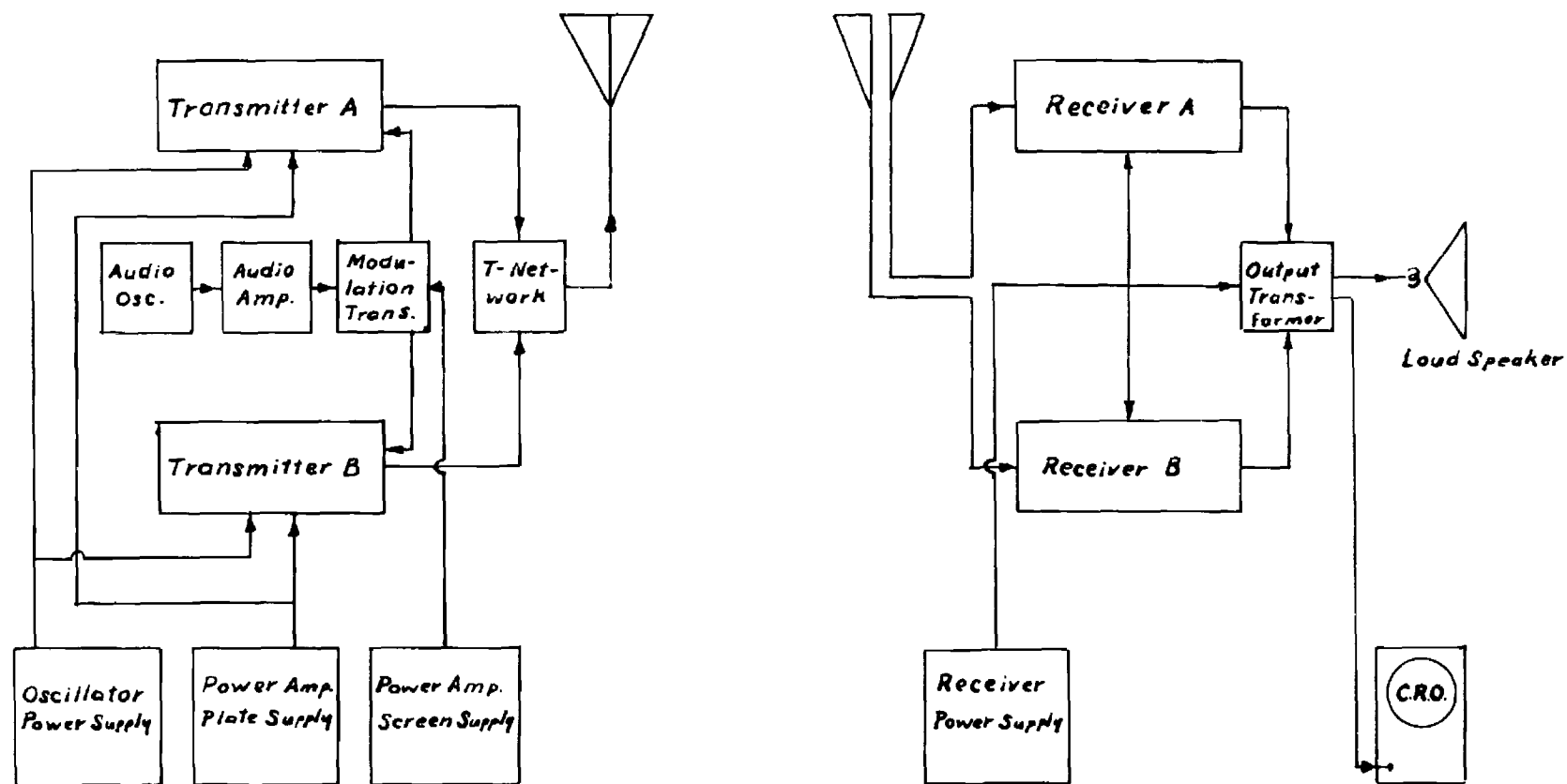


Figure 1. Block Diagram of Complete Two-Carrier Amplitude Modulation System As Used in the Experiment.

radio-frequency output of each was fed into opposite terminals of a T-network which permitted both transmitters to feed power into a common antenna and also served to dissipate most of their radio-frequency energy as heat. A piece of wire about ten feet in length extending outside the shielded room was used as an antenna, and the far end of the antenna was connected to a crystal detector which served as a monitoring detector for observing the transmitted signals simultaneously on an oscilloscope. Both transmitters were mounted on a metal chassis; the T-network was mounted on the under side of the chassis. Terminals for meters, power supply connections, modulation input, and the antenna were mounted on the chassis.

Each transmitter consisted of a 1626 (triode) master oscillator whose output was inductively coupled to the class "C" power amplifier, a pair of 1625 beam power tubes operating in parallel. Located in each transmitter unit was a 1629 electron ray tube (magic eye) which was used with a quartz crystal to give accurate visual indication of the master oscillator calibration. The power amplifiers were of the screen grid modulated type. Power for the oscillators, power amplifier screen grids, and power amplifier plates was furnished by three voltage-regulated direct current supplies.

The master oscillator consisted of a Hartley circuit with good frequency stability. Tuning was accomplished by means of a variable condenser in the oscillator tank circuit. This condenser was ganged with the power amplifier tuning condenser. Tracking of the two condensers was facilitated by means of a trimmer condenser in parallel with the oscillator tuning condenser and an adjustable iron slug in the oscillator coil. A temperature compensating condenser of low capacity was connected across the oscillator tuning coil to reduce the effect of temperature changes

upon the frequency of the master oscillator.

The electron ray tube in conjunction with a quartz crystal was used to calibrate the oscillator tuning dial accurately. Radio-frequency voltage was taken from a tap near the ground end of the oscillator coil and applied to the control grid of the ray tube. The shadow angle appearing on the face of the tube was maintained at a small value by means of heavy grid bias. When the oscillator was operating at the resonant frequency of the crystal, a radio-frequency voltage appeared across the crystal and hence was applied to the grid of the electron ray tube, thus causing the shadow angle to increase. The resonant frequency of the crystal was 6.2 mc, the mid-frequency point on the tuning dials.

The power amplifier utilized a pair of 1625 beam power tubes connected in parallel and operated in class "C" using grid leak bias. The grids received their radio-frequency excitation from a tapped pickup coil inductively coupled to the master oscillator tank coil. The tap was maintained at radio-frequency ground potential, and grid neutralization was employed to prevent the amplifier from oscillating. The power amplifier tuning condenser was ganged to the master oscillator tuning condenser. Additional capacity was provided by means of a variable condenser (adjusted initially and remaining fixed) in shunt with the tuning condenser. An iron slug was used in the tank coil to aid in tracking the power amplifier tuning condenser.

Radio-frequency power was taken from the power amplifier tank coil by means of a pickup coil of variable coupling. The load impedance (the input impedance of the antenna network) was resonated with a variable inductance in series with the pickup coil.

Amplitude modulation of the radio-frequency output was accomplished

by varying the voltage applied to the power amplifier screen grids at an audio-frequency rate. The direct current plate potential was maintained constant. Modulation of the two transmitters in phase opposition was effected by supplying the screen voltage for each power amplifier from opposite ends of the secondary winding of the modulation transformer. The direct-current screen-grid voltage was applied at the center tap of the secondary winding on the modulation transformer. Very nearly half-envelope modulation was obtained by reducing the direct-current screen voltage to zero; under this condition each transmitter supplied power to the antenna in slightly more than one-half of each modulation cycle.

Audio-frequency power for modulating the transmitters was supplied by an amplifier fed by a Hewlett-Packard audio oscillator.

Description of the Receivers. Two used, modified U. S. Army Signal Corps receivers, type BC 455-A, with an operating range of frequencies of 6.0 to 9.1 mc were employed. The receivers, receiver gain controls, output transformer, and power supply terminals were mounted on a metal chassis.

The receiving antenna consisted of a pair of insulated wires about six feet long twisted loosely together. Each of the wires was connected to the antenna terminal on a receiver.

The receivers were of the superheterodyne type. The antenna signal was received and amplified in a single-stage radio-frequency amplifier, and the output was applied to the hexode section of the mixer stage. The local oscillator operated at a frequency above the incoming signal frequency and applied its output to the hexode section of the mixer. The mixer output was applied to a two-stage intermediate-frequency amplifier.

A diode detector rectified the output of the intermediate-frequency amplifier, thus reproducing the transmitted audio-frequency signal of the respective transmitter. The detected signal was then amplified in a single-stage audio amplifier.

The input section of the receivers consisted of a tuned radio-frequency amplifier employing a 12SK7 pentode and a mixer stage employing a 12K8 tube with the triode section connected in an oscillator circuit. The antenna input circuit, the mixer grid circuit, and the local oscillator plate circuit were gang tuned by a three-section variable condenser. The coils tuned by the variable condenser were equipped with adjustable iron cores to facilitate the receiver alignment; trimmer condensers, also, were connected in parallel with the tuning condensers for this purpose.

The intermediate-frequency amplifier consisted of two stages employing type 12SK7 remote cut-off pentode tubes. Both stages employed cathode bias. This means of biasing was employed also in the radio-frequency amplifier stage. The amplification of the receiver was controlled by varying the resistance in the cathode circuits of the first and third stages of the receiver. No automatic volume control was employed.

The output of the second intermediate-frequency amplifier stage was applied to one diode section of a type 12SR7 tube. The second diode plate was not used and was connected to ground. The rectified output of the detector was applied through a coupling condenser to the input of the audio amplifier. A jack wired to the detector output was provided so that the rectified waveform could be observed on an oscilloscope. The triode section of the 12SR7 tube was connected in a beat-frequency oscillator

circuit and was used when receiving code signals. However, the oscillators were not used in the tests being discussed.

The audio amplifier consisted of a type 12A6 beam power tube operating as class "A". The plate of the tube was connected to one end of the center tapped primary winding on the output transformer, which was added for this work. The plate of the power-amplifier tube in the other receiver was connected to the opposite end of the primary winding. Direct-current plate voltage was applied at the center tap. It was in this way that the resultants of modulation frequency were combined differentially.

EXPERIMENTAL TECHNIQUES AND DATA

Objectives of the Experiment. The objectives of the experiment were as follows:

1. To show that noise components of the impulsive type appearing in the output of a receiver can be reduced by balance with the output of a similar receiver (adjusted to equal gain) which is tuned to a different frequency;

2. To illustrate the increase in receiver signal output in the half-envelope system (modulation method of Case 2) of amplitude modulation over that of the conventional amplitude modulation system with equality of total transmitted carrier power in the two systems being maintained.

Observed Noise Reduction. Both transmitters were placed in operation using carrier frequencies of 6.7 and 6.8 mc and modulated at a frequency of 1000 cycles. Full envelope modulation was employed, and both carriers were modulated at the same percentage. The power output of the transmitters was made equal. The gain control of receiver A (tuned to 6.7 mc) was turned to about two-thirds of its maximum position and left fixed. The signal output from the detector of this receiver was observed on an oscilloscope and the wave amplitude recorded. The detector output of receiver B (tuned to 6.8 mc) was adjusted to the same amplitude by varying the receiver gain control with the aid of the oscilloscope. The gain control setting of receiver B was recorded so that equal receiver gains at the specified tuning frequencies could be obtained without aid of the transmitters.

Noise known to be produced by nearby rotating electrical machinery was heard in the output of the receivers. The gain of receiver B was reduced to zero and then adjusted to equal that of receiver A. A substantial reduction in noise output was noticed as the gain of receiver B was increased from zero to the value giving balance of receiver gains, both in the presence of the transmitted signals and in their absence.

A noise generator consisting of a Ford (Model "T") ignition coil, spark gap, and an antenna about three feet long was placed some twenty feet from the receivers. Receiver B gain was set at zero. The noise voltage appearing across the secondary winding of the output transformer was observed with an oscilloscope. The gain of receiver B was adjusted for balance of gains, and a reduction of about ten to one in the average amplitude of the noise peaks was observed. This observation was made in the absence of transmitted carrier waves.

The output of a pulse generator was fed directly into the antenna terminals of the receivers simultaneously. Receiver A was tuned to 6.7 mc and receiver B was tuned to 6.8 mc as before. Pulses having an amplitude of 0.05 volt and repetition frequencies of 500 per second and 5000 per second were fed into the receivers. The duration of the pulses was varied over a considerable range. The purpose of this variation was to determine the pulse durations for a number of maxima and minima of the curve of Fourier coefficients (see Figure 7). Maximum and minimum output from each receiver did not occur at precisely the same values of pulse widths because of the slight difference in receiver center frequencies. Several widely differing values of pulse duration, each chosen to be as nearly equal to the values of duration for maximum output from

both receivers, were fed into the receivers simultaneously. These values of pulse duration were chosen so that the coefficients of the corresponding components intercepted by both receivers would be approximately equal. The voltage developed across the secondary of the output transformer was measured with a vacuum tube voltmeter (peak operated). The gain of receiver A was fixed. Readings were then taken for receiver B gain of zero and then for balance of gains as given in Table I.

TABLE I
OBSERVED CANCELLATION OF RECEIVED PULSES

Pulse Amplitude	Pulse Repetition Frequency	Pulse Duration	Output Gain _B = 0	Output G _B = G _A	Voltage Reduction Factor	Noise Reduction Decibels
0.05V	500/sec	1.07 μ s	6.15V	0.5V	0.081	22
0.05V	500/sec	10.2 μ s	6.2 V	0.9V	0.145	17
0.05V	500/sec	100.0 μ s	5.1 V	0.5V	0.098	20
0.05V	5000/sec	1.0 μ s	11.0 V	1.8V	0.164	16
0.05V	5000/sec	10.0 μ s	9.6 V	1.5V	0.156	16

Observed Increase in Received Signal. Transmitter A was placed in operation using a carrier frequency of 6.7 mc and a modulation frequency of 1000 cycles. A direct-current screen-grid potential was applied so as to make possible full envelope modulation, and the modulating signal was increased until a flattening of the negative peak of the reproduced signal was observed on the monitoring oscilloscope. This condition corresponds to 100% modulation of the transmitted radio-frequency envelope for a perfectly linear transmitter. The waveform observed on the oscilloscope was good, and no saturation effects near its positive peak was apparent. The waveform of the receiver detector output appeared to be

undistorted when observed on an oscilloscope. The power amplifier plate voltage, plate current, screen voltage, and screen current were read and recorded. The audio-frequency voltage output of the receiver was read in vertical scale divisions on the screen of the oscilloscope. The receiver gain and oscilloscope Y-amplifier gain remained fixed. Transmitter B was then placed in operation using a carrier frequency of 6.8 mc. Half-envelope modulation of the radio-frequency carriers was obtained by adjusting the power-amplifier direct-current screen-grid voltage to zero in both transmitters. The direct-current plate voltage of both power amplifiers was made equal to the voltage used previously in transmitter A. The modulating voltage was increased until the plate current drawn by each power amplifier equalled one-half that originally drawn by transmitter A which employed conventional amplitude modulation. Therefore, the total direct-current power supplied by the final amplifiers was equal to that previously supplied by the final amplifier of transmitter A. It is reasonable to assume that the total output of carrier frequency was also substantially equal. The gain of receiver B was then adjusted to equal that of receiver A, and the audio-frequency voltage output of the receivers was observed on the oscilloscope and found to be 1.5 times that read previously from receiver A. This increase in the output signal voltage corresponds to a power increase in the ratio of 2.25.

The readings taken in performing this final phase of the experiment are given in Table II.

TABLE II
COMPARISON OF RECEIVED SIGNALS

	Transmitter A	Transmitter B
1. Oscillator Plate Volts:	250 V	
2. P. A. Plate Current:	20 ma	
3. P. A. Plate Voltage:	250 V	
4. P. A. Screen Current:	17.5 ma	
5. P. A. Screen Voltage:	130 V	
6. Carrier Frequency:	6.7 mc	Not Used
7. Modulation Frequency:	1000 cycles	
8. Percentage Modulation:	100% (approx.)	
9. Receiver Output as read in 0.1" Divisions on Oscilloscope:	20 (crest to crest)	
1. Oscillator Plate Volts:	250 V	250 V
2. P. A. Plate Plate Current:	10 ma	10 ma
3. P. A. Plate Voltage:	250 V	250 V
4. P. A. Screen Current:	1.0 ma	1.0 ma
5. P. A. Screen Voltage:	0.0 V	0.0 V
6. Carrier Frequency :	6.7 mc	6.8 mc
7. Modulation Frequency:	1000 cycles	1000 cycles
8. Percentage Modulation:	Half-Envelope	Half-Envelope
9. Receiver Output as read in 0.1" Divisions on Oscilloscope:	30 (crest to crest)	

Thus it is seen that the observed values are in reasonable agreement with those calculated.

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APPENDIX

	PAGE
Photograph of Transmitter Units, Associated Meters, and Power Supplies.....	46
Photograph of Receiver Units, Bench Power Supply, Oscilloscope, and Monitoring Speaker.....	47
Schematic Diagram of a Transmitter Unit, Including Antenna Network.....	48
Schematic Diagram of a Receiver Unit, Including Transformer Used in Combining Signals Differentially.....	49
Table Showing Components of the Transmitters.....	50
Table Showing Components of the Receivers.....	51

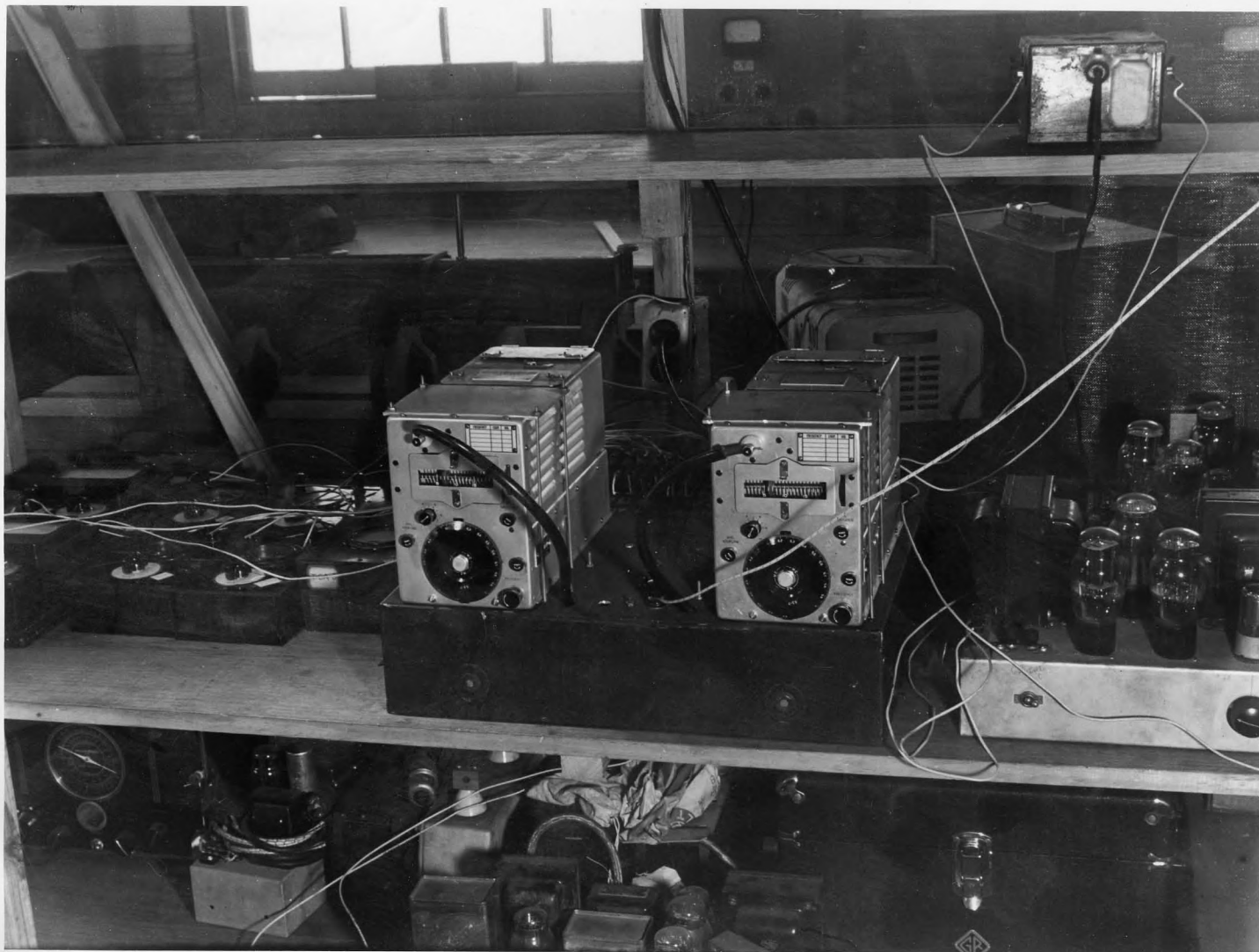


Figure 10. Photograph of Transmitter Units,
Associated Meters, and Power Supplies.

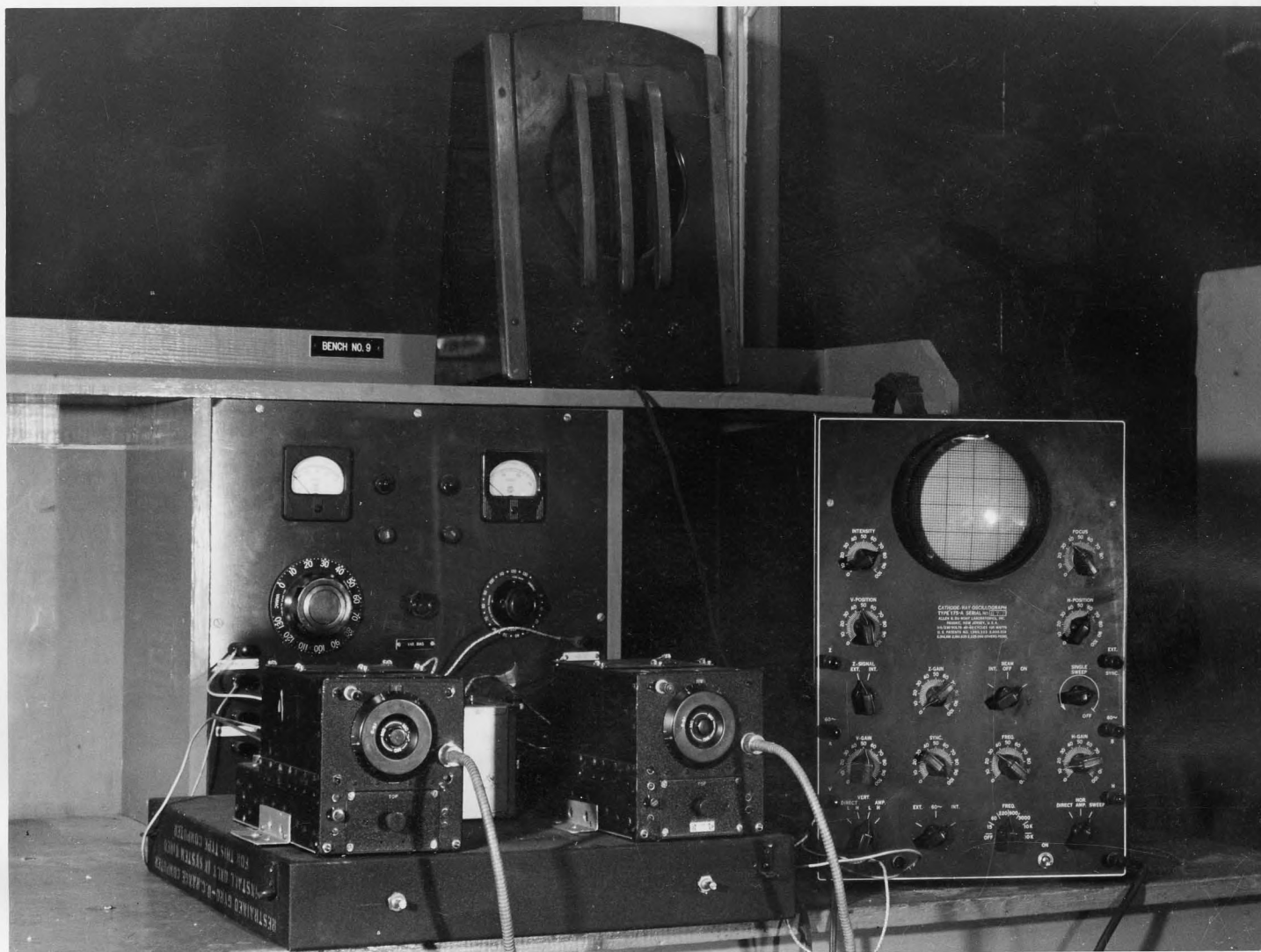


Figure 11. Photograph of Receiver Units, Bench Power Supply, Oscilloscope, and Monitoring Speaker.

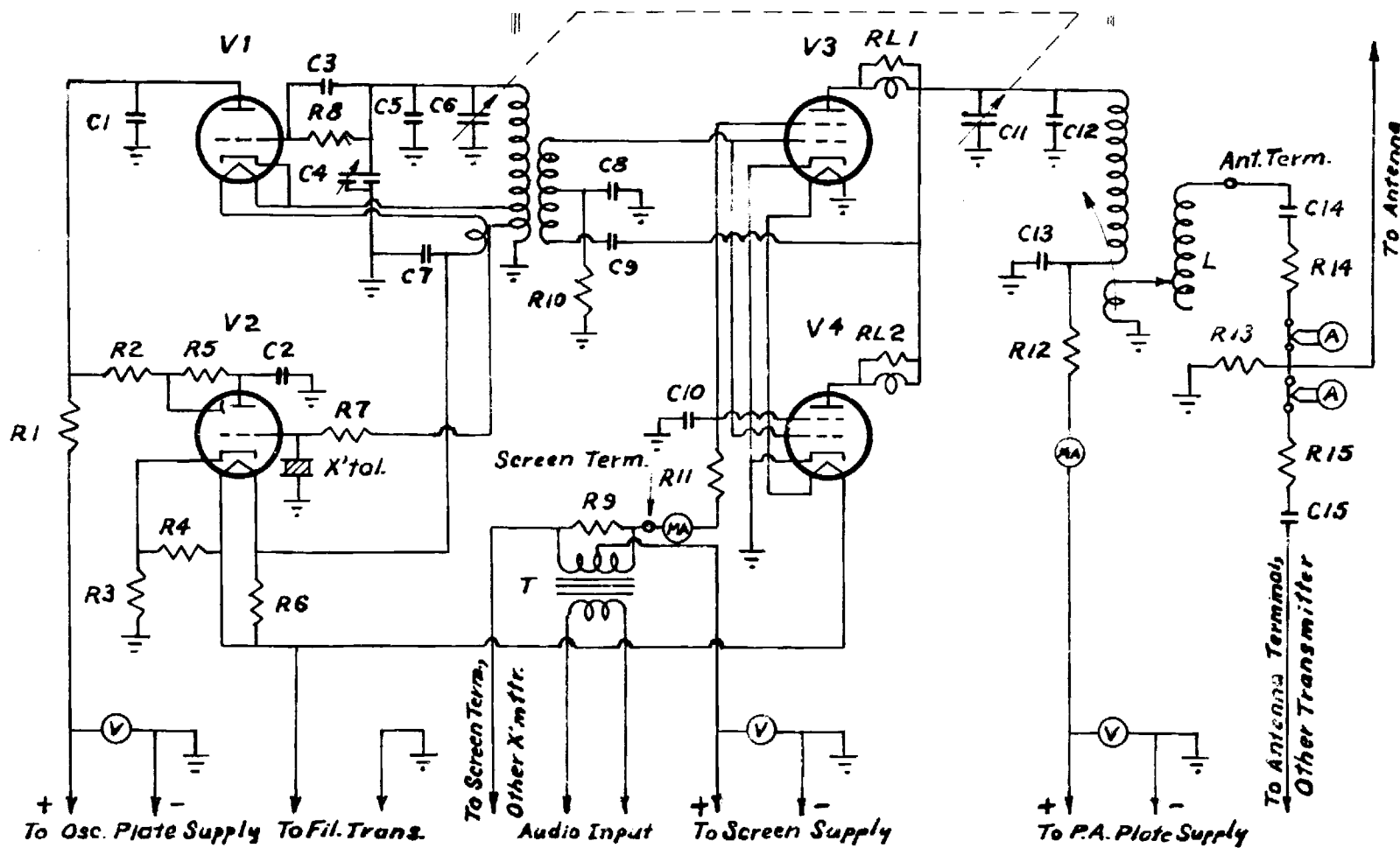


Figure 12. Schematic diagram of a Transmitter Unit, Including Antenna Network, with Block Connections to the Other Transmitter Unit.

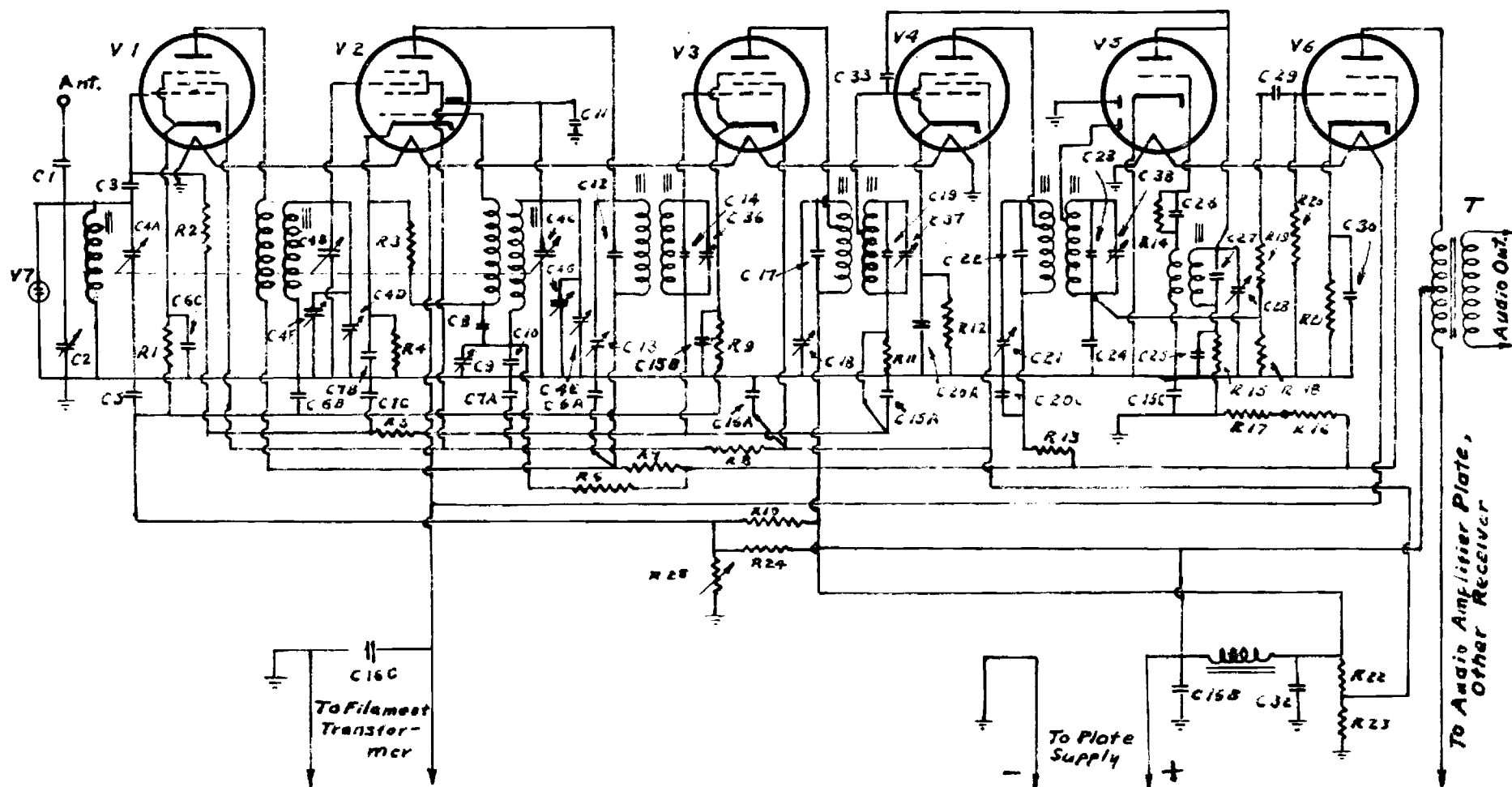


Figure 13. Schematic Diagram of a Receiver Unit, Including Transformer Used in Combining the Signals Differentially. High shows Connections to the Other Receiver Unit.

TABLE III
COMPONENTS OF THE TRANSMITTERS

CAPACITANCES		RESISTANCES		MISCELLANEOUS	
Symbol	Description	Symbol	Description	Symbol	Description
C1	.05 mfd.	R1	20 ohms	V1	1621 Master Osc.
C2	.05 mfd.	R2	51,000 ohms	V2	1629 Magic Eye
C3	.00018 mfd.	R3	390 ohms	V3	1625 Power Amp.
C4	M.O.Padding	R4	1000 ohms	V4	1625 Power Amp.
C5	.000003 mfd.	R5	1,000,000 ohms	L	Ant. Inductance
C6	M.O.Tuning	R6	126 ohms	T	Mod. Transformer
C7	.006 mfd.	R7	15,000 ohms		
C8	.05 mfd.	R8	51,000 ohms		
C9	Fixed Neut.	R9	15,000 ohms		
C10	.002 mfd.	R10	15,000 ohms		
C11	P.A.Tuning	R11	51 ohms		
C12	P.A.Padding	R12	20 ohms		
C13	.01 mfd.	R13	0.5 ohms		
C14	.00005 mfd.	R14	5 ohms		
C15	.00005 mfd.	R15	5 ohms		
		RL1	Parasitic Supp.		
		RL2	Parasitic Supp.		

TABLE IV
COMPONENTS OF THE RECEIVERS

CAPACITANCES		CAPACITANCES		RESISTANCES		MISCELLANEOUS	
Symbol	Description	Symbol	Description	Symbol	Description	Symbol	Description
C1	11 mmf.	C28	34 mmf.	R1	620 ohms	V1	12SK7
C2	15 mmf.	C29	.02 mfd.	R2	2,000,000 ohms	V2	12K8
C3	100 mmf.	C30	40 mfd.	R3	51,000 ohms	V3	12SK7
C4(A to G)	Gang(147mmf.)			R4	620 ohms	V4	12SK7
C5	3 mfd.	C32	5 mfd.	R5	150,000 ohms	V5	12SR7
C6(A,B,C)	.05/.05/.05mfd.	C33	Wiring Cap.	R6	200,000 ohms	V6	12A6
C7(A,B,C)	.05/.05/.05mfd.		(less than	R7	200 ohms	V7	Neon Lamp
C8	200 mmf.		2 mmf.)	R8	200 ohms	T	Output Transformer
C9	40 mmf.			R9	620 ohms		
C10	365 mmf.	C36	17 mmf.	R10	260,000 ohms		
C11	3 mmf.	C37	17 mmf.	R11	100,000 ohms		
C12	180 mmf.	C38	17 mmf.	R12	510 ohms		
C13	17 mmf.			R13	200 ohms		
C14	180 mmf.			R14	100,000 ohms		
C15(A,B,C)	.05/.05/.05mfd.			R15	20,000 ohms		
C16(A,B,C)	.22/.22/.22mfd.			R16	100,000 ohms		
C17	180 mmf.			R17	100,000 ohms		
C18	17 mmf.			R18	510,000 ohms		
C19	180 mmf.			R19	100,000 ohms		
C20(A,C)	.05/.05 mfd.			R20	2,000,000 ohms		
C21	.7 mmf.			R21	350 ohms		
C22	180 mmf.			R22	7000 ohms		
C23	180 mmf.			R23	7000 ohms		
C24	30 mmf.			R24	20,000 ohms		
C25	.001 mfd.			R25	2500 ohms		
C26	100 mmf.						
C27	180 mmf.						